Performance Analysis of Improved Noise Cancellation Discrete Hartley Transform ACO-OFDM in a AWGN channel

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***Abstract*—Asymmetrically Clipped Optical Orthogonal Frequency Division Multiplexing (ACO-OFDM) stands as one of the most promising modulation techniques in high-speed optical communications. However, enhancing the error rate or spectral efficiency of conventional ACO-OFDM in optical systems remains a challenge. Existing enhanced ACO-OFDM demodulation techniques often suffer from either high computational complexity or poor performance due to limitations like DC-offsets and low frequency noise. To address this challenge, we propose a novel ACO-OFDM demodulation scheme based on Hartley transform and Pulse Amplitude Modulation (PAM), named "Improved Noise Cancellation ACO-OFDM (INC ACO-OFDM)". This scheme combines the advantages of both Diversity-Combined ACO-OFDM (DC ACO-OFDM) and Noise Cancellation ACO-OFDM (NC ACO-OFDM) without employing Hermitian symmetry. The proposed INC ACO-OFDM technique uses a Virtual Clean Windows (VCW) identification process, allowing the use of even subcarriers of ACO-OFDM to enhance the demodulation performance, even in the presence of DC offset. Simulation results conducted using MATLAB R2021a over an Additive White Gaussian Noise (AWGN) channel demonstrate that the proposed INC ACO-OFDM technique achieves similar Bit Error Rate (BER) performance as DC ACO-OFDM with a significant reduction in computational complexity, providing a 3 dB power gain compared to ACO-OFDM with similar spectral efficiency.**

Keywords— Discrete Hartley Transform, Improved Noise Cancellation, Diversity Combining, AWGN, ACO-OFDM

I. Introduction

The exponential growth of services (high-speed internet, VoIP, cloud computing, the Internet of Things, etc.) in communication networks has led to the pursuit of a new alternative (broad bandwidth) capable of handling traffic and, particularly, high-speed data transfer. With existing infrastructures becoming obsolete due to the increase of new services, optical fiber emerges as a strong candidate capable of overcoming the limitations of copper in achieving high-speed data transfer. Despite the vast bandwidth capacity of optical fiber, various modulation techniques have been developed to optimize its use: single-carrier modulations and, notably, multi-carrier modulations such as OFDM (Orthogonal Frequency Division Multiplexing). In conventional OFDM systems, modulation and demodulation of orthogonal subcarriers are performed using Discrete Fourier Transform (DFT) and Inverse DFT (IDFT), respectively [1]. These processes are accelerated and made cost-effective by the availability of numerous Fast Fourier Transform (FFT) and Inverse FFT (IFFT) algorithms, as well as advancements in field-programmable gate arrays (FPGA) [2]. Despite these benefits, OFDM systems face challenges such as a high peak-to-average power ratio (PAPR), which necessitates RF amplifiers to operate over a wide linear range [3]. Additionally, OFDM is highly sensitive to frequency offsets and channel temporal variations, which can disrupt the orthogonality of subcarriers and cause Inter-Carrier Interference (ICI). As a result, OFDM systems require precise synchronization between the transmitter and receiver [4]. Numerous studies have explored methods to address these issues and enhance OFDM system performance. One method that has garnered significant attention in recent years is the substitution or combination of the DFT block in conventional OFDM systems with alternative orthogonal transforms include the Discrete Cosine Transform (DCT) and Discrete Sine Transform (DST), the Walsh-Hadamard Transform (WHT), the Wavelet Packet Transform (WPT) and the Discrete Hartley Transform (DHT). The aim is to enhance system performance—such as reducing the Peak-to-Average Power Ratio (PAPR) or minimizing bit error rates—or to simply reduce computational complexity [5]. The DHT, in particular, is notable for its real-valued nature, requiring only real arithmetic operations. This makes it especially suitable for real-valued signals, such as those used in Binary Phase Shift

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Keying (BPSK) modulations and optical communication systems [6]. Furthermore, the DHT and its inverse (IDHT) share an identical kernel, allowing for the use of the same circuitry in both the transmitter and receiver. This interesting property will receive our attention in this paper.

In optical communications, OFDM has several variants, including DCO-OFDM (DC-biased Optical OFDM), ACO-OFDM (Asymmetrically Clipped Optical OFDM), and others [7]. Among these techniques, ACO-OFDM stands out as a promising approach offering significant advantages in terms of power efficiency and robustness against interference. However, improving the bit error rate or spectral efficiency of ACO-OFDM in optical systems remains a challenge. In the literature, ACO-OFDM is reported to have the same data rate as FLIP-OFDM and U-OFDM [8]-[9]. Asymmetrically clipped DC-biased optical ADO-OFDM is the first hybrid technique that combines aspects of both DCO and ACO-OFDM. While ADO-OFDM maintains the same data rate as DCO-OFDM, it demonstrates poorer bit error rate (BER) performance compared to the aforementioned methods [10]-[11]. The asymmetrically and symmetrically clipped optical ASCO-OFDM technique is another hybrid approach that achieves a higher data rate than ACO-OFDM and FLIP-OFDM, while keeping a comparable BER performance to DCO-OFDM. More details about these O-OFDM schemes can be found in papers [8]-[12].

Moreover, to address the limitation in ACO-OFDM, the unmodulated even subcarriers can be considered as a new channel with its own set of odd- and even-indexed subcarriers. A new ACO-OFDM layer can then modulate the odd subcarriers of this new channel and overlay them on the existing ACO-OFDM signal without causing interference. This process can be recursively repeated, resulting in a bandwidth-efficient approach known as layered ACO-OFDM [12]-[13]. It is widely recognized that the conventional ACO-OFDM receiver, which utilizes only the odd subcarriers and disregards the even ones, is suboptimal [14]. The even subcarriers contain information that can assist in the symbol detection of the odd subcarriers [15]. Then, several enhanced ACO-OFDM receivers have been proposed, demonstrating significant BER performance but at the cost of increased complexity performance over the conventional receiver [16]. A detailed explanation of these receiver schemes is provided in papers [17]-[18]. It is evident that most of them are affected by either high computational complexity or poor performance due to reception noise or DC offset [18]-[19].

This paper demonstrates that replacing the DFT block with a DHT block in ACO-OFDM schemes, using a new demodulation approach, can lead to significant improvements in system performance and a reduction in computational complexity, even in the presence of DC offset and noise. The proposed technique, called "Improved Noise Cancellation ACO-OFDM (INC ACO-OFDM)," combines the benefits of DHT and IDHT with both Diversity-Combined ACO-OFDM (DC ACO-OFDM) and Noise Cancellation ACO-OFDM (NC ACO-OFDM) [19]-[20], without relying on Hermitian symmetry [21].

II. METHODS

In this section, we present the general principles and technical aspects of ACO-OFDM, along with the proposed optical improved ACO-OFDM method.

A. Conventional ACO-OFDM Modulation

In ACO-OFDM, data symbols are transmitted on odd subcarriers, while even subcarriers are used to create a bias signal. This bias ensures that the resulting OFDM signal remains non-negative, meeting the requirements for optical transmission. The input vector $X = [0, X_1, 0, \ldots, X_{N-1}]$ to the IFFT block of size N must satisfy the Hermitian symmetry property, as defined in (1). A cyclic prefix is appended to the resulting time-domain signal x, which is then clipped at zero. Clipping noise [19] affects only the unused even subcarriers, but not the odd subcarriers. The resulting signal is real and adheres to the anti-symmetry property defined by (2).

$$X_k = X^*_{N-k} \quad for \quad 0 < k < \frac{N}{2} \tag{1}$$

$$x_k = -x_{k+\frac{N}{2}} \quad for \quad 0 < k < \frac{N}{2} \tag{2}$$

At the receiver, the optical signal is first converted back to an electrical signal. The subsequent processing follows the same steps as in a conventional OFDM receiver [9]. Fig. 1 illustrates the block diagram of a standard ACO-OFDM system as described earlier.

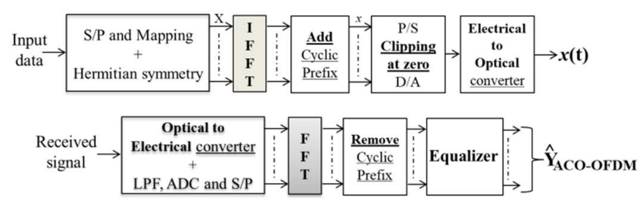

Fig. 1. Conventional ACO-OFDM block diagram.

B. Discrete Hartley Transform (DHT) ACO-OFDM

The block diagram of the ACO-OFDM system based on DHT is shown in Fig. 2. The IDHT and DHT are used instead of conventional IFFT and FFT to perform the OFDM modulation and demodulation, respectively. According to the DHT function [21]-[22], the OFDM symbol is given by (3), where N is the IDHT size, k = 0, 1, 2,…, N-1, and *x(n)* the n^{th} symbol sequence.

$$h(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) \left[\cos\left(\frac{2\pi kn}{N}\right) + \sin\left(\frac{2\pi kn}{N}\right)\right] \tag{3}$$

Similarly, to return to the frequency domain, a DHT transform is applied as described by (4):

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} h(k) \left[\cos\left(\frac{2\pi k}{N}\right) + \sin\left(\frac{2\pi k}{N}\right)\right] \tag{4}$$

In DHT ACO-OFDM (Fig. 2), only odd subcarriers are used for data transmission to generate a structured signal so that direct zero-clipping (without adding a DC component) is possible without information loss [22]. In other words, the odd components of an ACO symbol contain useful information, while the even components are defined by zeros. At transmitter side, the odd subcarriers are modulated via an IDHT block. The resulting bipolar signal is then clipped to zero, thus producing a unipolar DHT ACO-OFDM signal. At the reception, the steps for the transmitter are executed in reverse order. It is important to note that despite the absence of a DC component, the signal is correctly demodulated. This is due to the antisymmetric property [21] of the ACO-OFDM illustrated by (4), meaning that each positive sample (k) has a negative sample at $(k+\frac{N}{2})$ with the same amplitude.

$$x(k) = -x\left(k+\frac{N}{2}\right), \qquad 0 < k < \frac{N}{2} \tag{5}$$

C. Noise Cancellation DHT ACO-OFDM

The Noise Cancellation DHT ACO-OFDM (NC DHT-ACO) is an improved variant derived from the DHT ACO-OFDM [19] using an identical transmitter. The only difference lies in the demodulator (Fig. 3), where a "Noise Cancellation" block is implemented in the received time-domain signal. Due to the antisymmetric property (5) of the DHT ACO-OFDM signal, the Noise Cancellation process can identify the samples of antisymmetric pairs most susceptible to noise and forcing them to zero. After the Noise Cancellation process, the rest of the method is similar to the conventional DHT ACO-OFDM demodulation [19]-[20]. However, occasionally the wrong sample can be set to zero in presence of noise or DC-offset. Such errors can lead to a degradation in system performance.

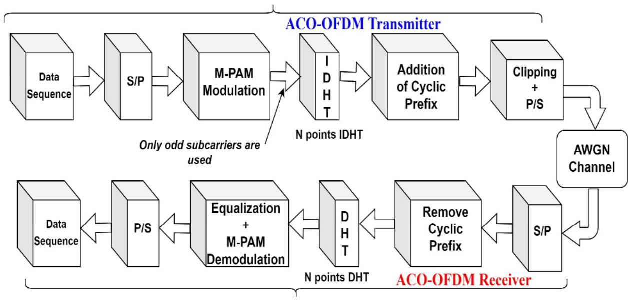

Fig. 2. Conventional ACO-OFDM block diagram.

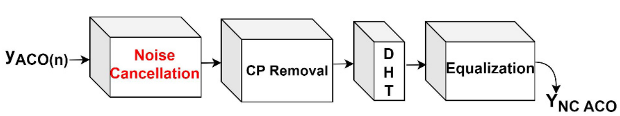

Fig. 3. Block diagram of the DHT NC-ACO.

D. Diversity-Combined DHT ACO-OFDM

The goal of the Diversity-Combined DHT ACO-OFDM (DHT DC-ACO) system, is to enhance the performance of the DHT ACO by leveraging both odd and even-order subcarriers during reception. Both DHT ACO and DHT DC-ACO use the same transmission scheme where only odd-order subcarriers are used for information transport. At reception, the received and equalized signal Y_{ACO} undergoes a process where it is divided into its odd and even components: Y_{even} and Y_{odd}. The Y_{odd} component is then processed using an IDHT to obtain y_{odd}, which represents the information from the odd-order subcarriers [23]-[24]. This helps to estimate the DC-offset (6), denoted as $Y(0)$ which is crucial for accurate demodulation and signal reconstruction [20].

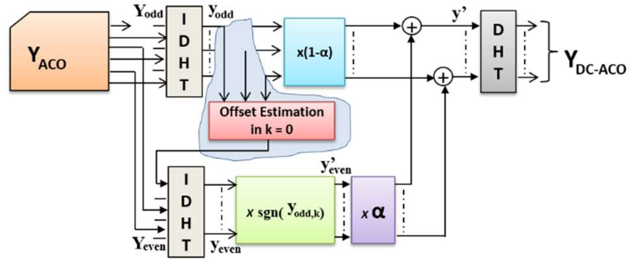

Fig. 4. Block diagram of the DHT DC-ACO.

$$Y(0) = \sum_{k=0}^{N-1} |y_{odd}(k)| \tag{6}$$

Fig. 4 shows the detailed steps of this process, including the block diagram of the DHT DC-ACO OFDM demodulator. The even component, with the zeroth subcarrier replaced by $Y(0)$, is fed into another IDHT block, producing an output signal y_{even}. The absolute value of y_{even} is then multiplied by the sign of y_{odd} to reconstruct a bipolar signal, denoted as y'_{even}. The combined signal y' is then obtained using (7), where α is a weighting factor [19].

$$y' = (1-\alpha)y_{odd} + \alpha y'_{even} \tag{7}$$

Finally, a DHT is applied to yield Y_{DC-ACO}, from which the data is recovered in the same manner as in the conventional DHT ACO system.

E. DHT Improved Noise Cancellation ACO-OFDM

It has been demonstrated that applying Noise Cancellation and Diversity-Combining processes to ACO-OFDM maintains system performance, but at the cost of increased computational complexity [25].

Given the relatively high computational complexity of DHT DC-ACO, practical issues such as increased hardware costs and higher energy consumption may arise. To address these concerns, the DHT Improved Noise Cancellation ACO-OFDM (DHT INC-ACO) is proposed in this paper as an alternative. This technique aims to achieve at least the same demodulation performance as DHT DC-ACO but with reduced computational complexity. The DHT INC-ACO technique implements the Noise Cancellation algorithm after the conventional DHT ACO-OFDM equalization. It incorporates the DC-Offset estimation using a Virtual Clean Windows (VCW) concept. Due to clipping at the transmitter, the DHT

ACO-OFDM signal contains many zeros. On the receiver side, these zeros create a virtual "clean" window that allows for the observation of degradation and noise without interference from the data signal. To better understand the VCW principle, it is crucial to note that in the conventional noise cancellation method, all non-zero amplitudes in the VCW are reset to zero. If there is no DC offset, the demodulation performance of the DHT NC-ACO approach can seemingly improve demodulation performance by effectively canceling a large amount of noise. However, in the presence of noise or DC-offset, wrong decisions can be made. To solve this problem, a VCW is employed. As illustrated in Fig. 5, after equalization in the conventional DHT ACO-OFDM, the equalized symbols Y_{ACO} are then entered into an IDHT block, which yields to a time-domain signal y_{ACO} expressed as (8). Here, x_{ACO} represents the discrete DHT ACO-OFDM signal transmitted through the channel. d and w_1 correspond respectively to the DC-offset and the noise of the received signal. According to the antisymmetric property (5) and zero-clipping operation, the identification of the VCW can be simplified to a series of paired detections between two hypotheses [19] as follows (9)-(10):

$$y_{ACO}(k) = x_{ACO}(k) + d + w_1(k) \tag{8}$$

$$H_0 : \begin{cases} y_{ACO}(k) > 0 \\ y_{ACO}(k+N/2) = 0 \end{cases} \quad k = 0,1,\dots,N/2-1 \tag{9}$$

$$H_1 : \begin{cases} y_{ACO}(k) = 0 \\ y_{ACO}(k+N/2) > 0 \end{cases} \quad k = 0,1,\dots,N/2-1 \tag{10}$$

Based on these established hypotheses H_0 and H_1, the decision criterion is given by (11):

$$y_{ACO}(k) - d \underset{H_1}{\overset{H_0}{\gtrless}} y_{ACO}\left(k+\frac{N}{2}\right) - d \tag{11}$$

Thus, with (11), the VCW can be expressed as a set of indices $I \triangleq \{n_i,\ for\ i = 0,1,\dots,N/2-1\}$ where (12):

$$n_i = \begin{cases} i + \frac{N}{2} & if\ H_0\ is\ true \\ i & if\ H_1\ is\ true \end{cases} \tag{12}$$

Once the positions of I are identified through the VCW process, the DC offset, denoted as dc, can be estimated using (13) with $e(k)$ the estimation error whose power is as large as $4/N^2$ of that of $w_1(k)$:

$$dc = \frac{2}{N}\sum_{i=0}^{\frac{N}{2}-1} y_{ACO}(n_i) = d + e(k) \tag{13}$$

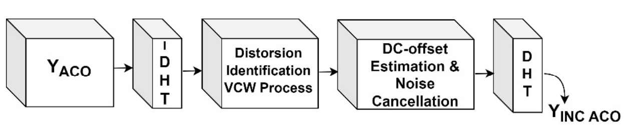

Fig. 5. Block diagram of the DHT INC-ACO.

With a sufficiently large value of N, the estimation error $e(k)$ becomes negligible, allowing d to be approximated as the true DC-offset. Once the DC-offset value is obtained, it can be canceled from the noise in $y_{ACO}(n)$ using (14)-(15):

$$y_1(k) = \begin{cases} y_{ACO}(k) - dc & if\ k \in \bar{I} \\ 0 & if\ k \in I \end{cases} \tag{14}$$

$$y_2(k) = \begin{cases} y_1(k) & if\ \ y_1(k) \geq 0 \\ 0 & if\ \ y_1(k) < 0 \end{cases} \tag{15}$$

Hence, data can simply be recovered after extracting symbols from the odd subcarriers of $y_2(k)$.

As a result, the DHT INC-ACO reduces the computational complexity of DHT DC-ACO by eliminating one IDHT block. In the following section, we will evaluate the performance of the proposed approach in an AWGN channel.

III. NUMERICAL RESULTS

In this section, we present the results from simulations conducted in an AWGN channel. Each OFDM symbol in our approach is generated with an IDHT/DHT size of N=512 using 2~16-PAM mapping, compared to the IFFT/FFT block used in conventional OFDM methods. A cyclic prefix (CP) of 128 samples is employed. The digital OFDM signal is generated using MATLAB R2021a. Bit Error Rate (BER) performance is assessed through Monte Carlo simulations [23].

Different performance metrics are utilized in the literature to compare modulation schemes in optical systems. One common metric is the optical signal-to-noise ratio (SNR), which measures performance based on the ratio of optical power to the standard deviation of zero-mean noise power in the optical system. Another metric is the electrical energy-per-bit to single-sided noise power spectral density ratio, $E_{b(elec)}/N_0$, which is also used to assess performance. Additionally, the effective SNR, defined as the ratio between OFDM signal power and effective noise power, is sometimes employed for performance comparison. Note that, as $E_{b(elec)}/N_0$ increases, the SNR also increases. In this paper, we use $E_{b(elec)}/N_0$ to compare BER performances in an AWGN channel, following the approach used in [6].

Fig. 6 and Fig. 7 summarize the results of our study. We can see that the BER improves as SNR increases. This can be explained by the fact that as the SNR increases, the transmitted signal becomes less affected by channel noise, resulting in more accurate symbol transmission.

A comparative study between conventional ACO-OFDM and DHT ACO-OFDM techniques is shown in Fig. 6. It can be seen that for any M-PAM constellation size, the DHT-ACO technique exhibits similar BER performance as conventional M^2-QAM ACO-OFDM modulation. This can be explained by the fact that the spectral efficiency in a M-PAM constellation is equal to spectral efficiency of a M^2-QAM constellation with Hermitian Symmetry. Since each constellation transmits the same number of bits per symbol, it is clear that the emitted M-PAM or M^2-QAM symbols experience a similar level of noise impact. Moreover, at a fixed $E_{b(elec)}/N_0$, we observe that the

BER deteriorates as the constellation size increases, whether in M-PAM DHT ACO-OFDM or conventional M²-QAM ACO-OFDM. This explains why, to transmit more bits (for example, moving from 2-PAM to 4-PAM or 16-PAM), a sufficiently high $E_{b(elec)}/N_0$ is required to ensure the same BER. Since the DHT and IDHT use the same kernel, employing identical circuitry in both the transmitter and receiver simplifies the design and reduces power consumption. Consequently, we demonstrate that M-PAM DHT ACO-OFDM can transmit the same bit sequence at the same data rate as conventional M²-QAM ACO-OFDM, but with lower power consumption.

Fig. 7 presents a comparative study of DHT INC-ACO versus DHT DC-ACO, focusing on $E_{b(elec)}/N_0$ and comparable spectral efficiencies. It can be observed that both techniques achieve similar BER performance, with an approximately 3 dB gain in SNR compared to conventional ACO-OFDM (Cf. Fig. 7 versus Fig. 6). This improvement is due to the effective demodulation provided by both Diversity-Combined ACO-OFDM and Improved Noise Cancellation ACO-OFDM at the receiver. These advanced demodulators reduce the system noise impact by approximately half.

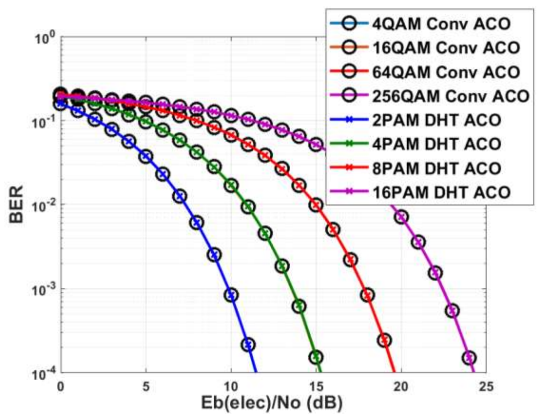

Fig. 6. BER performance of DHT ACO-OFDM vs ACO-OFDM as a function of $E_{b(elec)}/N_0$.

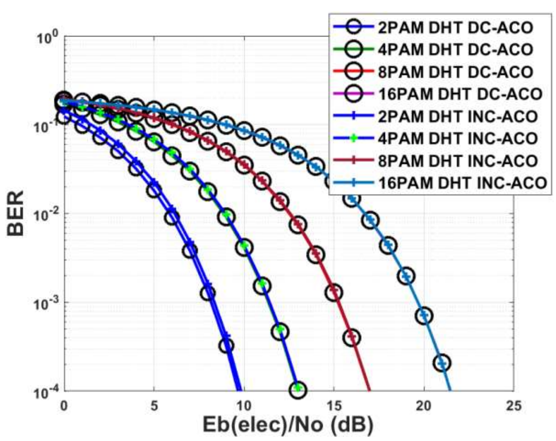

Fig. 7. BER performance of DHT INC-ACO vs DHT DC-ACO as a function $E_{b(elec)}/N_0$.

Since DHT INC-ACO reduces the computational complexity of DHT DC-ACO by removing one IDHT while preserving the same BER performance, it stands out as a promising candidate for improved ACO-OFDM modulation in optical systems.

IV. CONCLUSION

The DHT INC ACO-OFDM system presented in this paper appears to be a very interesting and innovative scheme. Optical fiber systems using IM/DD would benefit from this solution. The proposed approach analyzed in an AWGN channel shows a 3 dB power gain compared to conventional ACO-OFDM with low computational complexity and merits an intensive research in a realistic condition like optical fiber transmission with multi-layers aspects or error corrections.

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